Parallel Minimum Spanning Tree Algorithms

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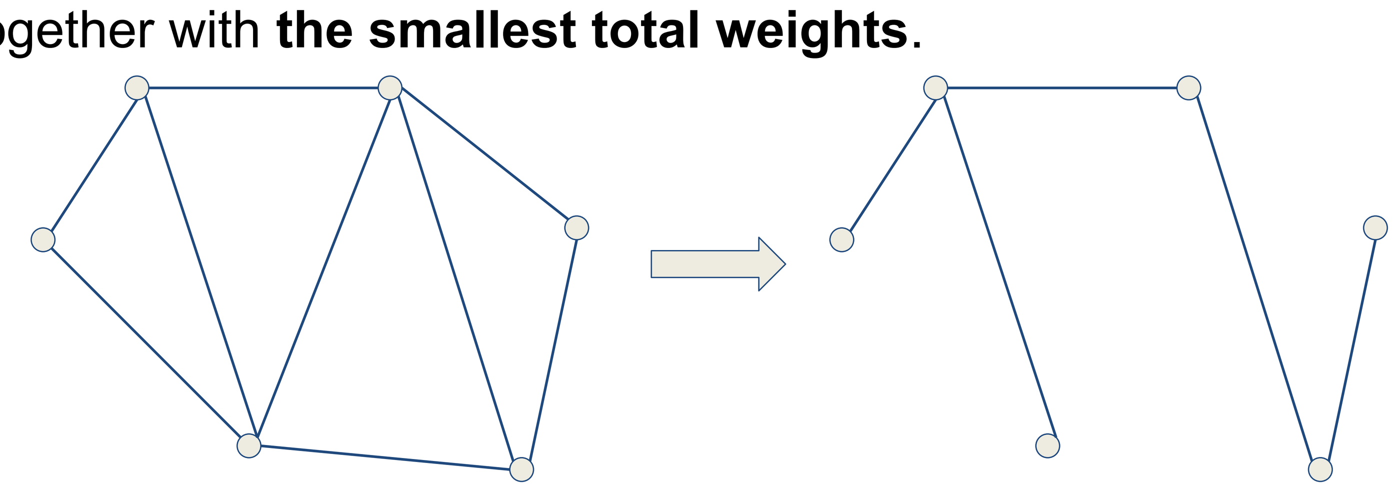
1. Problem

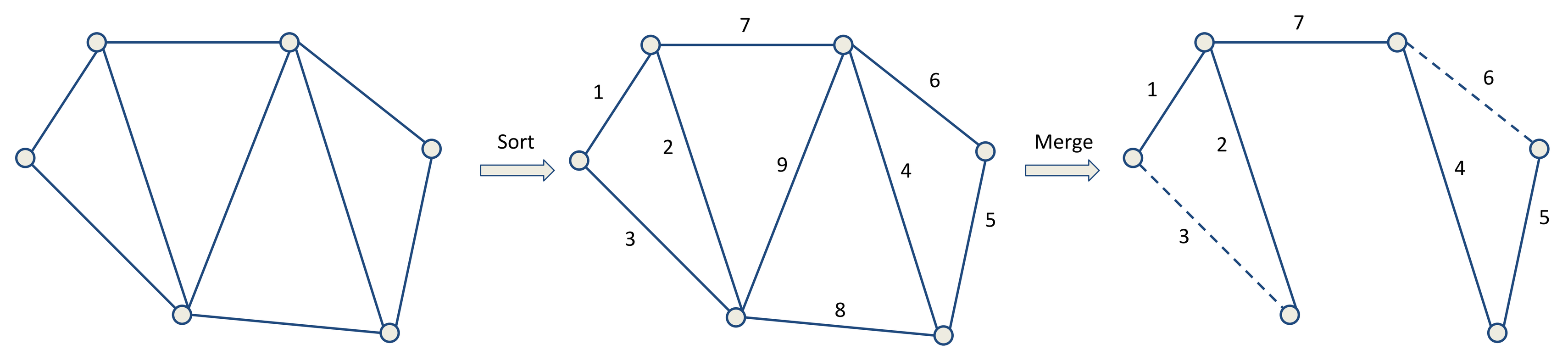
- In a connected, edge-weighted undirected graph, a minimum spanning tree (MST) is a subset of edges that **connects all nodes** together with **the smallest total weights**.



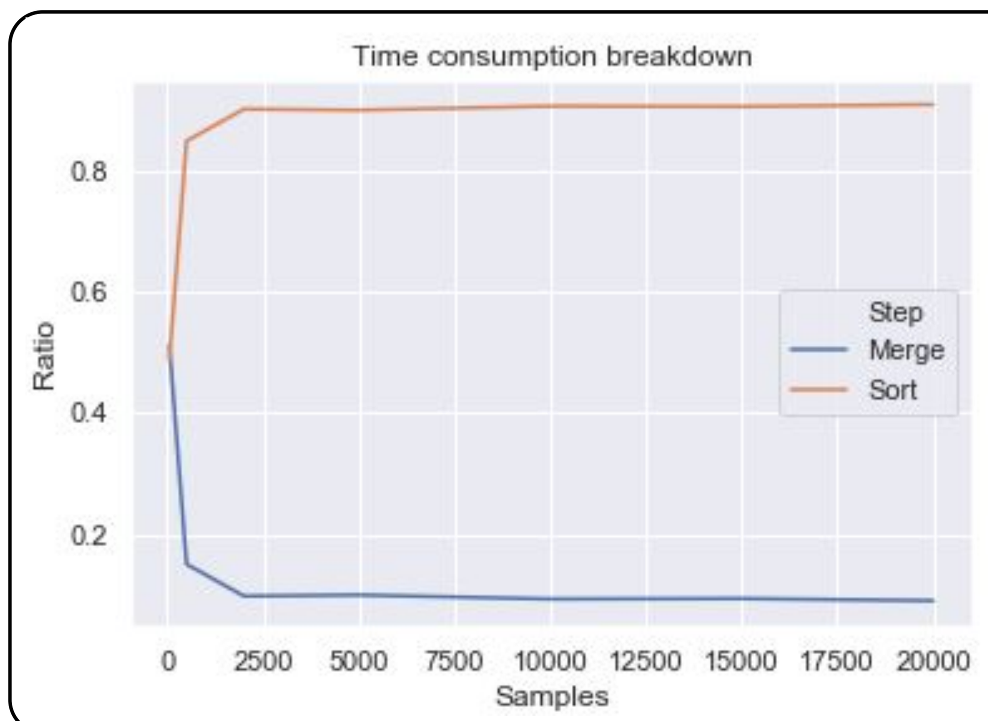
- Well-studied, Prim's algorithm [1] and Kruskal's algorithm [2]
- Prim's algorithm is hard to parallelize because each step depends on the sub-graph built previously.

2. Kruskal's Algorithm

- Kruskal's algorithm
  - Sort: sort all edges by weights
  - Merge: add the smallest edge that does not create a cycle
  - Time complexity: $O(e \log e)$ in sorting, and $O(e)$ in merging



3. Serial Algorithm and Profiling



Conclusion:

In large-scale dataset, sorting takes the majority of the time.

4. Proposed Parallel Algorithms

We use OpenMP framework to parallelize sorting via different methods.

- Enumeration Sort (not evaluated)
  - For each value, count the number that is smaller than it
  - Parallel in nature but has $O(n^2)$ complexity
- Parallel Quick Sort
  - Partition the list, and recursively sort two individual partition
  - Parallelize recursive calls via OpenMP tasks
- Sample Sort [3]
  - Select k pivots to partition the dataset, and then sort each partition individually via OpenMP loop parallelism
  - Use oversampling to balance the workload

5. Dataset

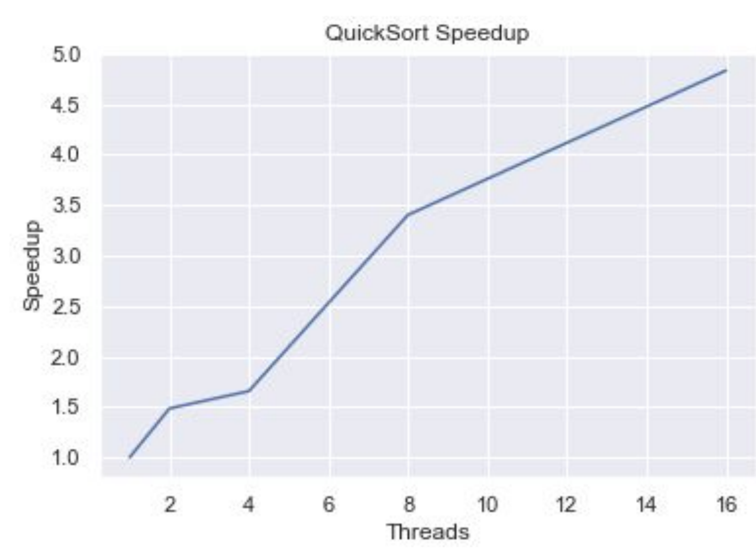
The minimal spanning tree of a input graph is decided by the order of edge weights and the graph topology.

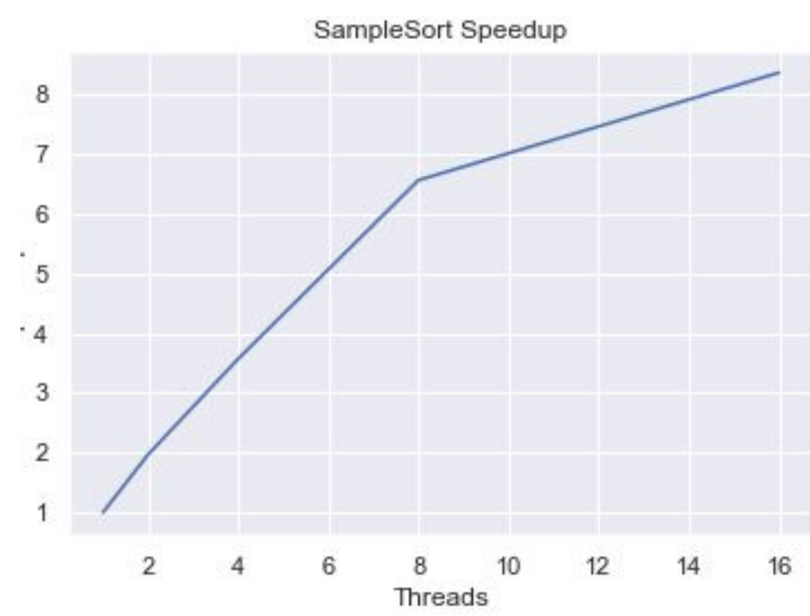
- Number of vertices: 40000.
- Weight: generate a random edge permutation by uniformly sampling the weight between 1.0 and 10.0.
- Topology:
  - Random graph: For any two distinguished vertices u and v, edge (u, v) is in the graph with probability $p \in \{0.01, 0.05, 0.1\}$.
  - Sparse graph: For any vertex u, $d \in \{10, 50, 100\}$ distinguished vertices v are connected to u. After removing duplicated edges, the degree of each vertex is bounded between d and $2d$.
  - Power-law graph: The vertex degree d satisfies the power-law distribution $p(d) \sim d^{-r}$, $r \in \{1.5, 2.5, 3.5\}$. Each vertex u is connected with d distinguished vertices v and duplicated edges are removed.

6. Experiments & Results

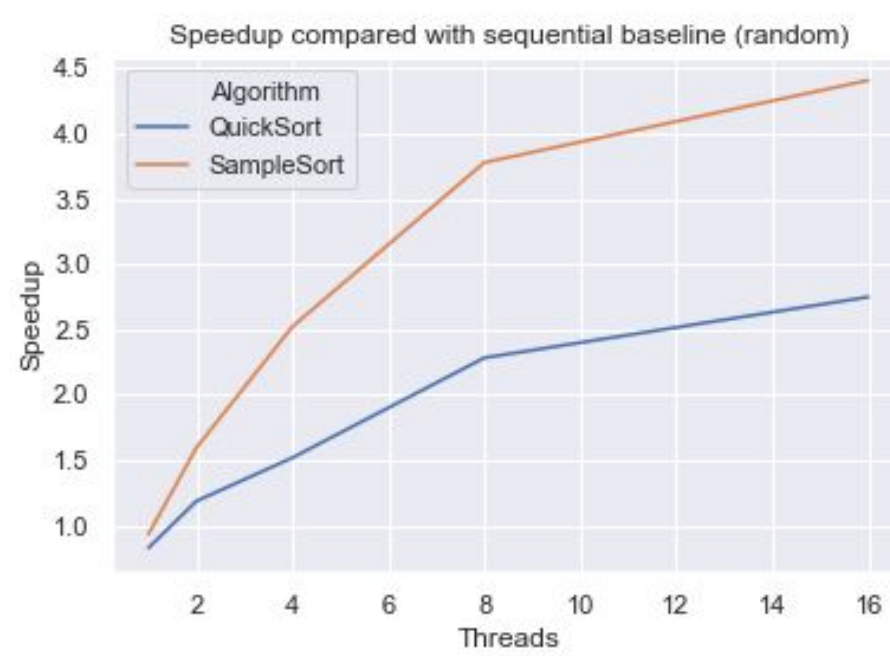
Experiments are conducted on GHC machine 41 with a 8-cores Intel(R) Xeon(R) E5-1660 v4 @ 3.20GHz processor and 32GB RAM.

Sorting Algorithm Speedup:

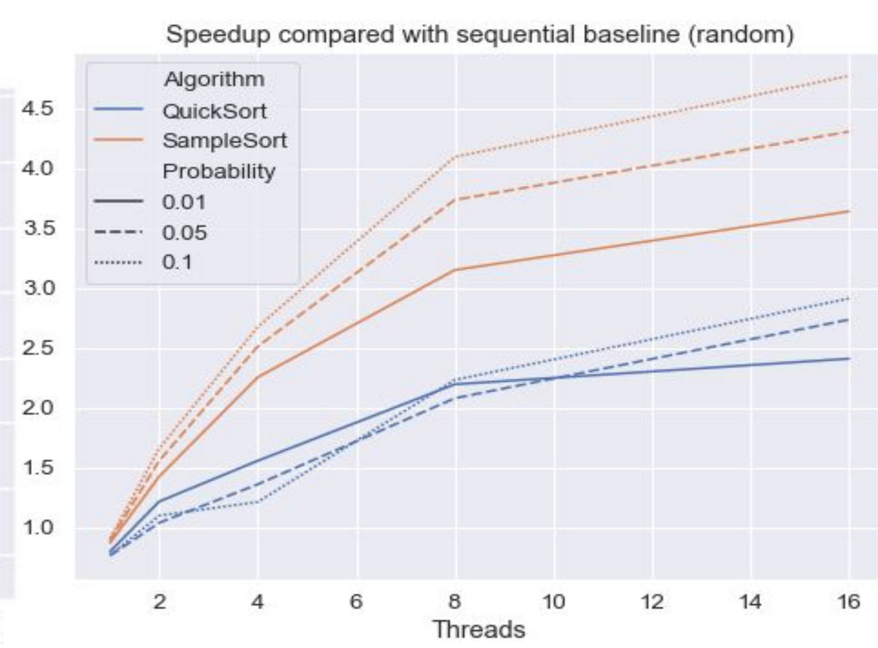




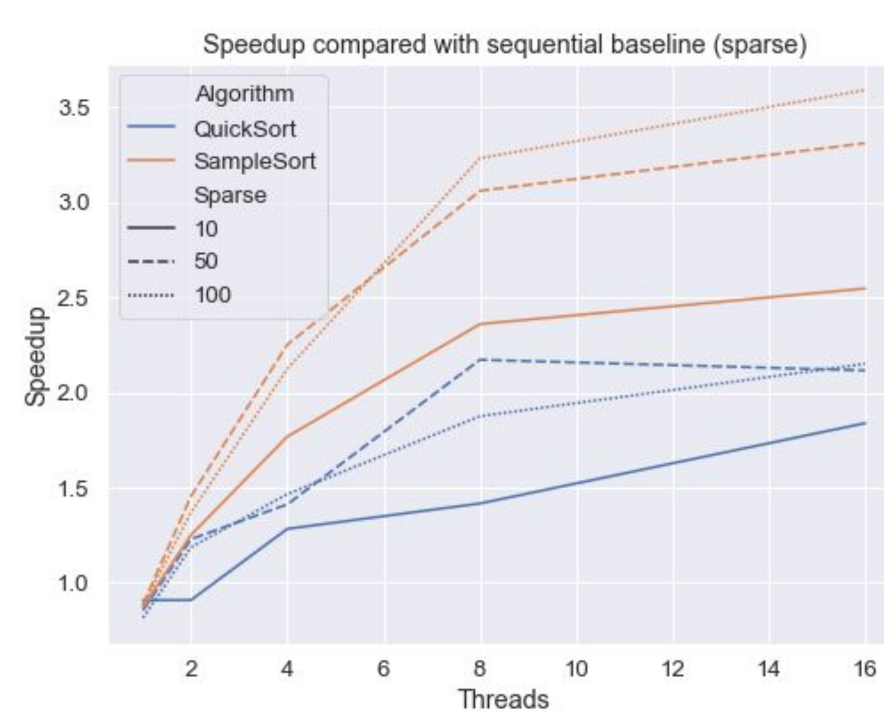
Parallel Minimum Spanning Tree:

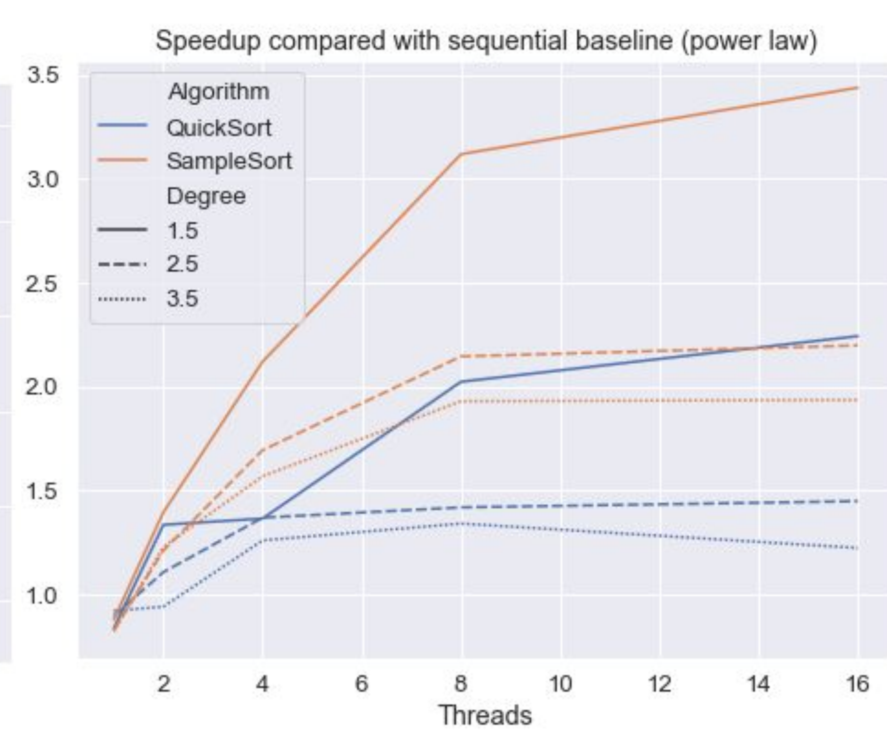


Speedup for Different Graphs:



Speedup for Different Graphs (Cont'):





[1] R. C. Prim, "Shortest connection networks and some generalizations," The Bell System Technical Journal, vol. 36,no. 6, pp. 1389–1401, 1957.
[2] J. B. Kruskal, "On the shortest spanning subtree of a graph and the traveling salesman problem," Proceedings of the American Mathematical society, vol. 7, no. 1, pp. 48–50, 1956.
[3] N. M. Amato, R. Iyer, S. Sundaresan, and Y. Wu, "A comparison of parallel sorting algorithms on different architectures," Technical Report TR98-029, Department of Computer Science, Texas A&M University, 1996.